THE MATHEMATICS TEACHER

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INTRODUCTORY COURSE IN MATHEMATICS.

BY DAVID EUGENE SMITH.

In spite of all that has been said in this country in opposition to mathematics in the past few years, the feeling of certainty still exists in the intellectual world that the science is not dead, is not dying, and is not stagnant; that it touches more lines of human interest to-day than ever before; and that its values have only been accentuated by the efforts made to relegate it to the position of formal grammar, formal rhetoric, and formal logic. Indeed, it may safely be said that mathematics stands more firmly to-day than ever before, not only in the minds of what is commonly called the intellectual class but in the opinions of the man in the shop and of the man who has so recently been in the trenches on the battlefields of France.

There are various reasons for this improvement of the position of mathematics. Some of these reasons are concerned with the character of certain criticisms which have been advanced, and with some of the strange, not to say wild, suggestions which have been made for improvement in the teaching of the subject. The man of real scholarship and the man of intellectual leadership is certain to look upon many of the criticisms as puerile, and the teacher who has made a success in his profession will as surely look upon many of the suggestions for improvement as the strange vagaries of men who are wholly ignorant of the problem. In particular, the criticism that algebra has nothing that the average citizen needs is one that has

materially strengthened the position of the subject among thoughtful students of education, while the suggestion that all mathematics should be blended without reference to the radical differences of purpose of certain branches, or that it should be taught only by some project method, has made the successful teacher suspicious of the aid to be expected from the professor of education.

As a matter of fact, the discussion has brought to the front the values of a subject like algebra; it has shown more clearly the distinctive purposes of branches like algebra and geometry; and it has shown that there is no more reason for making mathematics merely an adjunct to the work of the shop, which is often the essence of the project method, than there is for abolishing physical education except as it is an adjunct to manual work on the farm or in the factory.

Teachers of mathematics have been accused of standing back of the shield of tradition, and the same may be said of the church and of our most valued social customs; they have been charged with failing to state with definiteness the precise purposes of their courses, and the same may very likely be said of every course in education in every school of education in this country; they have been said to desire the use of public funds for that which is not practical, and the same may be said of every course in music, the fine arts, or *belles lettres* everywhere.

But out of all this dispute, often running into the ridiculous or the pitiful, there has come considerable good. I am inclined to think that mathematics is rated higher throughout the country, and that the science of education, or at least some portion of its retainers, has been rated lower. This I infer from careful reports covering a wide range of our territory.

The teachers of mathematics have undoubtedly been spurred to greater activity, and this is generally a good thing. They have been urged to examine their problem more thoroughly, to encourage the elimination of those features which have lost their essential character, to compare our work with the work done in other countries, to see how the subject can be made more real to the pupil, to determine what new features can be introduced to replace those which are eliminated, and to do all this by adding positive values which no fair mind would question.

As a result of this searching of the soul of our science and of our own souls as teachers there has come one decided change in our work, and for this we have reason to thank the school administrators. I refer to the movement that would extend the high school down to the seventh school year. It is immaterial what name we give to the school which begins in the seventh grade, whether we speak of the Junior High School as covering grades seven, eight, and nine, or of the "six and six plan," or of departmental teaching in the grades; the essential thing is that we have in this movement the possibility of introducing mathematics in a more rational way than the one heretofore followed by us, and to embody in our work the best that the rest of the world has to offer in the teaching of the subject. Of all the advanced countries of Europe and America, and we may also include Japan, we have been unquestionably the most backward in our introductory work in mathematics; but at last the opportunity has come for us to profit by the experience of other nations and by the results of our own investigations into the needs of our people.

The bases upon which we should rest our new structure are unquestionably the needs of our people in the home, in various industries, and in commercial and other activities. Whatever of disciplinary values mathematics has,—and it is interesting to see the diversity of view of those who so vainly seek to prove that it has none,—this disciplinary value will be developed as effectively from mathematics resting upon those foundations as from a science which, in its first stages of presentation, is based solely upon considerations of abstract logic.

If we recognize, as most of the world does, that the utilitarian operations of arithmetic have been essentially covered at the end of the sixth school year, we still have to teach those general applications which everyone needs to know but which the child's experiences do not enable him to grasp in the earlier years. At the same time we have to make sure that the child's powers of computation do not atrophy through lack of use, a thing that is sure to happen if arithmetic is entirely stopped at that time.

If we have three years of a Junior High School, therefore, we have this general proposition to consider: What shall be the

extent, the nature, and the position of arithmetic in order that its general applications shall best be understood and the habit of accurate computation retained and strengthened?

Without answering this question at present, we may consider the position of geometry and algebra. A few years ago there was some local agitation in the country in favor of fusing, as it was called, the work in algebra with that in geometry. Failing in this, there was the effort to make demonstrative geometry a branch of applied mathematics, or at least to seek applications for its various propositions. Neither of these movements succeeded, and for very sound psychological reasons. In the first place, the aims and methods of demonstrative geometry are as distinct from those of algebra as are the aims and methods of chemistry or physics. There are a few points of contact, and various interesting analogies, but the demonstration of a proposition in geometry is undertaken for an entirely different purpose and by an entirely different method from those found in the solution of a linear equation, even though the latter has its analogue in the science of form. Mathematics is a traditional name; it does not happen to cover music but it does cover the art of computation, although in the Greek civilization music was included and the art of computation was not. Simply because geometry and algebra are both called mathematics, by present fashion, is no reason why two such essentially different subjects should be forced into a most unnatural wedlock.

Furthermore, the idea of searching for a series of practical applications for every proposition of demonstrative geometry is to lose entirely our clearness of vision. Let it be distinctly affirmed for the comfort of the educational iconoclast,—asking due pardon for resorting to the *argumentum ad populum,*—let it be distinctly said that demonstrative geometry is not, never was, and never will be taught because of its immediate practical applications. No workman in a shop ever applies any proposition in geometry that he could not apply just as well, if it were told to him, without proving it. He knows the Pythagorean Theorem before he studies geometry, he knows how to use it, and he also knows the fundamental property of similar figures. If the immediate application of the proofs of the propositions of demonstrative geometry to money making is the purpose of

the science, then demonstrative geometry must *ipso facto* cease to exist. It is on this account that the search for applications of any considerable number of demonstrations has been abortive,—and the demonstration, not the nearly obvious geometric fact, is the essence of geometry as we commonly understand the terms. One might as properly seek for a practical application of every painting in an art gallery, of every symphony of an orchestra, of every beauty in nature, and of every noble thought that stirs the soul.

Just as the word "arithmetic" has entirely changed its meaning in the last four centuries, however, and just as the name "algebra" has assumed a new significance within the same period, and just as the word "geometry" as completely changed its meaning after the time of Thales, so we are meeting with desirable changes to-day. To have students begin their work in demonstrative geometry of the older type is to lead a certain non-intellectual group into a hopeless maze. To have them begin their algebra on the plan of twenty years ago is equally unreasonable. No one believes in this method at the present time for the mixed class of students who enter our high schools, and all teachers of any promise earnestly wish to break away from it. We have happily reached a time when we can extend our concept of geometry and allow it to cover the intuitive as well as the demonstrative phase of the subject. Similarly, we have reached a time when we are safely extending our ideas of algebra to include a line of practical and interesting applications which render the introduction to the science far more psychological than anything that was known a generation ago.

And finally, by the way of introduction, let it be said that we have come to realize that algebra and geometry do not constitute all that there is of secondary mathematics. To realize what a rich field we have, it is only necessary to look into the work of countries like France or England and see how naturally a simple trigonometry fits into the early courses.

With these preliminary observations, I venture to suggest certain principles which should, I believe, guide us in planning the introductory work in mathematics.

First, arithmetic should occupy the pupil's attention in the first half of the seventh grade, so that the break in mathematics

shall not seem too pronounced. It should not be mere drill in computation, however; it should consist of those lines of application for which he was not prepared in his earlier work, and should be based upon real social needs,—his relation to the home, the store, and industry, and including the need for keeping accounts and avoiding waste.

This being done, the second half year may best be given to intuitive geometry. We have unconsciously recognized this, in a small way, by putting some mensuration into our arithmetic in the later years of our elementary school. The subject is, however, more extensive than that. The human mind naturally approaches geometry with three needs to be satisfied. Given an object, we may ask three questions of a geometric nature: (1) What is its shape? (2) How large is it? (3) Where is it? There are no others, for questions of value, color, odor, or use are not geometric in nature. Here, then, is the domain of intuitive geometry, a domain whose bounds are fixed by psychology, but one whose broad extent includes a large number of fields of immediate practical application. After these questions have been answered, there still remains the question of proving the correctness of the statements, and this is the domain of demonstrative geometry which the student may enter a year or so later.

In this domain of intuitive geometry the student learns the immediately practical side,—how to describe the shapes of objects, how to measure any objects that he is likely to need to measure in ordinary life, and how to locate points on a pattern, in the field, on a map, or in space, for each of which the number of genuine applications is very great. In all the work in measurements the student meets the formula in a natural way, he comes to know its meaning and its value, and thus he makes a beginning in algebra before the name has come to have any significance to him.

The next step is also determined by psychological considerations. The pupil has already seen some use for algebra, he has applied simple formulas in his intuitive geometry, and he is now ready to enter the new field. In making this entrance psychology again directs the way by telling us to continue to build upon the formula as we have already begun to do in Grade VII.

Since the pupil has a background of applied arithmetic and of mensuration, we should use the accumulated material as a basis for further formulas, thus correlating the geometry and algebra in the most effective manner. The student has now a reason for manipulating these formulas for the purpose of deriving his own rules, and so he comes to the equation. This, however, is not the equation of the older school, interesting as that puzzle was to most students and profitably as it may therefore be used as drill material; it is now the real equation, introduced naturally and used with a definite purpose. Following this work there naturally comes the graph of the formula, and thus the graph becomes something more than a mere picture of statistics of population of a territory or of the attendance in a school. If the graph is a curve, this curve may drop below zero, and hence the negative quantity enters naturally and tangibly. In a half year, therefore, the student comes to know the nature of and the uses for the four great features of elementary algebra,—the formula, the equation, the graph, and the negative number. He may know nothing of algebraic addition, or of any other operation; he may not know what a polynomial is; but he knows some of the great things of algebra, and he has accumulated a few practical tools which he can use in his reading about the simpler laws of mechanics and industry.

What does psychology now suggest? Naturally that the student should at once apply his algebra to arithmetic as well as to mensuration. To meet this natural demand there may now be introduced those wider ranges of business arithmetic for which the student's maturity of experience in mathematics and in his relation to life has prepared him. Such topics as the arithmetic of trade, of industry, of the bank, of corporate business, of daily life, of relations to the State, and of thrift and investments may well make use of the tools which algebra has furnished.

The student will thus come to know the chief uses of intuitive geometry and of the first part of elementary algebra. If he leaves school at the end of Grade VIII, he takes with him the immediately practical facts that he will need in his general reading and in such special walks of life as he is likely to enter.

If, however, the student proceeds in his school course,—then

what? It seems reasonable to give him some chance of knowing what real mathematics is like. We cannot yet tell what this new field will mean to him. He may have succeeded in the intuitive stage and fail here; the teacher may have failed to arouse his interest there, while here he may find that approach to exact truth which may stimulate him to worthy achievements. It seems criminal, therefore, to close to him the opportunity of trying himself in the larger domain; in other words, a subject which touches such a wide range of human interests and which offers the only knowledge of deductive logic that the school has as its command, should be made known to every student. This means the requiring of mathematics in the ninth school year.

What should be the nature of this mathematics? While intuitive geometry comes before algebra, being the more tangible, concrete subject, demonstrative geometry belongs late in such a course as this for the reason that it requires considerable maturity of judgment. We may therefore say that, in general, the first steps in the post-intuitive mathematics should be (1) algebra, (2) trigonometry, (3) demonstrative geometry. The algebra may properly include the fundamental operations, justified by constant reference to arithmetic, a subject upon which it can and should pour a flood of light; simple equations with practical applications; and an introduction to quadratics, with applications to those formulas likely to be met in the reading of books and papers about such subjects as automobiles, airplanes, and domestic science. Whether the student can factor an expression, or handle an awkward fraction is of little moment at this stage; the important thing is that he shall see what algebra means and shall be encouraged to continue the study if he has any gift in this direction.

After this introduction to the science of algebra the student should come to know the meaning of trigonometry, and he should learn how to use a tangent or a sine for practical purposes. Trigonometry does not naturally follow demonstrative geometry; it naturally follows intuitive geometry and algebra. It is chiefly an algebraic science based upon geometric intuition.

Finally, as an introduction to mathematics, it is the right and privilege of every student to know what demonstrative geometry means. It is here that many students first awaken to the

significance of mathematics. The appreciation of a demonstration and the contact with exact truth,—the value of these experiences in the adolescent period cannot be overestimated, and the educator who would bar a student from the possibilities of such experiences assumes a heavier burden than I should wish to carry.

It may be said that the student will not know much mathematics after such a course. This is true, and it is equally true that he does not know much mathematics after he completes the ninth school year at present. What he will know, however, is what three important parts of mathematics are about; he will know certain very important uses of the subject; and he will have tried himself out. If it is said that this smattering will dull the edge of interest, the answer is that every student who enters the École normale supérieure or the École polytechnique of France has gone through a similar smattering, and yet no schools in the world stand higher in mathematics. The teachers of mathematics have nothing to fear from such an introduction; the ones who have cause to fear are those who would eliminate from our high school courses mathematics as a serious subject.

This, to my mind, constitutes the proper approach to mathematics. It is also, to my mind, the minimum course to be offered and the maximum amount to be demanded. The student has now weighed his mental powers in the balance; he has shown whether or not he may profitably proceed with a subject which may carry him into the purest realm of thought, into the most profound speculations, or into the most important applications of science. Thereafter his work in mathematics should be the result of careful, sympathetic conference with his advisers at home and in the school. He may well stop at this point, or he may proceed farther, and the school should set no limit upon the extent of his progress provided there is sufficient demand to make the formation of classes justifiable. The educator who says that we, the teachers of mathematics, must show the value of mathematical astronomy, for example, should first show us the value of their own courses on the one hand, or of art, music, and the love of one's fellows on the other hand.

What I ask for mathematics I would ask for every branch of knowledge that touches such a wide range of human interests,—

art, science, language, *belles lettres,* history;—that every student in our high schools shall come to know what these subjects mean, how they touch and have touched humanity, and whether or not he or she is fitted to and cares to enter upon the serious study of one or more of them. In this way we shall improve our teaching, we shall do our duty to the youth of our country, and we shall stimulate the real study of a science in which we have a faith that does not fail us.

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A MESSAGE TO COLLEGE STUDENTS.

Do not get caught in the receding tide of the great war. Set yourselves at once to look forward. Remember that the world must be built up again, and it looks as if there was an opportunity to make the world better than it has ever been before. We believe there is a chance of preventing this thing from ever happening again, of building up mankind to something nearer a perfect condition, where every man can use his own faculties to the utmost, which, after all, is the great pleasure in life; where every man who has a heart and an ambition will be able to develop himself for something worth doing. Remember that, and look forward, you fellows that are young. Do not look back into the receding wave, but look forward into the crest that is coming on ahead of you. As in this war, so in civil life—your own right hand will teach you terrible things if you will only make your own right hand strong and use it for the right purpose, and begin now at once.—*President A. Lawrence Lowell, Harvard University.*

FIRST-YEAR ALGEBRA, AS DEVELOPED IN THE ACADEMIC HIGH SCHOOL, NEW BRITAIN, CONN.

By Robert R. Goff.

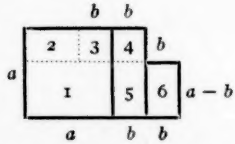
The aim of first-year algebra is to develop: (1) Ability to translate problems into equations of the first and second degree, and to solve these equations; (2) habits of systematic effort. For this purpose a study of the following topics is necessary:

1. The four operations, with much drill on signs, coefficients, and exponents.
2. Special products, with emphasis on the square of a binominal.
3. Factoring without grouping. Three cases:
   (1) Common monomial factor.
   (2) Quadratic trinomial.
   (3) Difference of two squares.
4. Fractions, principally with monomial denominators.
5. Linear equations.
6. Square root of arithmetical numbers.
7. Quadratic equations, solved by:
   (1) Graphs.
   (2) Factoring.
   (3) Completing the square.
8. Simultaneous linear equations, solved by addition or subtraction, and graphs.
9. Simultaneous linear-quadratic equations, solved by substitution, and graphs.
10. Problems,—ten types.

All of these topics are illustrated by geometrical figures. For example:

Factor $a^2 + ab - 2b^2$

Algebra	Geometry
$a^2 + ab - 2b^2 = (a + 2b)(a - b)$	Let $a =$ ——— ; $b =$ —



What figure represents a^2? ab? $2b^2$? $a^2 + ab - 2b^2$?
What does figure $2 =$? (6)
By rearranging, what rectangle is formed? $(1 + 5 + 6)$.
The sides of this rectangle are the factors.

The commoner definitions are shown by contrast.

Sum is the result of adding quantities.	*Product* is the result of multiplying quantities.
Term is one of the quantities of a sum.	*Factor* is one of the quantities of a product.
Coefficient shows how many times the quantity is used as a term.	*Exponent* shows how many times the quantity is used as a factor.
Multiple is the sum of identically equal terms.	*Power* is the product of identically equal factors.
Aliquot part is one of the identically equal terms of a multiple.	*Root* is one of the identically equal factors of a power.
Denominator shows how many equal aliquot parts.	*Index* shows how many equal roots.

The ten types of problems with their formulas are:

1. Equality of abstract related numbers.	$A +, -, \times, \div x = b +, -, \times, \div y$.
2. Cost, selling price, gain or loss.	$C + G = S$; $C - L = S$.
3. Mensuration (rectangles, etc.).	(*a*) $p = 2l + 2w$.
	(*b*) $a = lw$.
4. Number with digits.	$100h + 10t + u = n$.
5. Value of coins.	$25q + 10d + c = s$.
6. Changed number and price.	$np = (n + a)(p + b)$
7. Joint work.	(*a*) $1/a + 1/b = 1/c$.
	(*b*) $x/a + y/b = 1$.
8. Interest.	(*a*) $I = PRT/100$.
	(*b*) $A = P + PRT/100$
9. Mixtures.	(*a*) $ax + by = c(x + y)$.
	(*b*) $am = c(m + y)$.
10. Uniform motion.	$D = RT$.

Under class 6 are the lever problems where p is the power and n is the distance from the fulcrum.

In class 7, formula (*a*) is used when all the elements work all the time; formula (*b*) when some of the elements do not work

all the time. Under this case are the problems about the consuming of provisions by a number of men; also the filling of tanks by pipes.

In class 9, there are two common kinds of mixture problems:

(*a*) By taking from an *a* per cent. mixture and a *b* per cent. mixture to make a *c* per cent. mixture.

(*b*) By dilution, to change the per cent. of an ingredient in a mixture from *a* to *b*.

The second is a special case of the first when *b* equals zero.

Alloy problems come under class 9.

In class 10, uniform motion problems are of many types. Some of these are:

1. Two objects moving in the same direction for the same distance but unequal rates and times.

2. Two objects moving in opposite directions for the same time but unequal rates and distances.

3. An object moving with and against an outside force.

Rotating wheel problems often come under the first type; clock problems under the second.

The recitation is divided roughly into three parts:

1. Drill in analysis, recognition of types, and determination of method on past work; sometimes "chalk and talk."

2. Drill of a similar nature in the assignment for the next day.

3. Supervised study, with individual attention that the drill is followed.

New Britain, Conn.

INDETERMINATE FORMS IN TRIGONOMETRY.

By M. O. Tripp.

The subject of indeterminate forms is usually treated in algebra or in calculus; and hence it seldom occurs to the student that these forms may occur in trigonometry. It is sometimes desirable to examine critically some of the operations in proving identities, so that a false conclusion may not be drawn. Frequently the process of dividing both numerator and denominator by a factor holds only for certain values of the variable, or variables, involved. The occasional presence of indeterminates in solving spherical triangles teaches us that a spherical triangle is not always determined when three parts are given, as is sometimes supposed.

As an illustration of the danger involved in not being on the lookout for indeterminates, let us try to prove the identity

$$(1+\tan\theta)(1+\cot\theta)\sin\theta\cos\theta=(\sin\theta+\cos\theta)^2.$$

This can be rewritten in the form

$$\left(1+\frac{1}{\cot\theta}\right)(1+\cot\theta)\sin\theta\cos\theta=\sin^2\theta+2\sin\theta\cos\theta+\cos^2\theta,$$

or

$$\frac{1+2\cot\theta+\cot^2\theta}{\cot\theta}\cdot\sin\theta\cos\theta=\sin^2\theta+2\sin\theta\cos\theta+\cos^2\theta.$$

Clearing of fractions,

$$\sin\theta\cos\theta+2\sin\theta\cos\theta\cot\theta+\sin\theta\cos\theta\cot^2\theta$$
$$=\sin^2\theta\cot\theta+2\sin\theta\cos\theta\cot\theta+\cos^2\theta\cot\theta.$$

Simplifying and transposing,

$$\sin\theta\cos\theta-\sin^2\theta\cot\theta=\cos^2\theta\cot\theta-\sin\theta\cos\theta\cot^2\theta.$$

Factoring on each side,

$$\sin\theta(\cos\theta-\sin\theta\cot\theta)=\cos\theta\cot\theta(\cos\theta-\sin\theta\cot\theta).$$

If we now cancel the factor $(\cos\theta - \sin\theta\cot\theta)$ on each side of the equation, we get

$$\sin\theta = \cos\theta\cot\theta,$$

a result which is obviously not true for all values of θ. The difficulty is that the factor $(\cos\theta - \sin\theta\cot\theta)$ is always zero; so that, in reality, the cancelling of this factor is the same as taking $0/0$ as unity.

In the proving of identities students frequently make the mistake of assuming that they have proven an identity true when, by operating in the same way on each side of a supposed identity, they obtain an equation known to be true. Whereas, the real proof consists in starting from the equation known to be true, and proceeding to the supposed identity, step by step. For example, let us try to prove that

$$a = b.$$

Multiplying each side by 0, we get

$$a \cdot 0 = b \cdot 0,$$

a relation which is true; but we cannot pass from the latter relation to the former, because we encounter the indeterminate form $0/0$.

In passing from a supposed identity to a known identity, the supposed identity is proven true only in case every operation in the transformation is uniquely reversible. Suppose, for example, we try to prove that

$$a = -a.$$

Squaring

$$a^2 = a^2;$$

but from this we cannot assume that

$$a = -a,$$

since square root is not the unique reversal of squaring.

If we transform the equation

$$\tan(x+y) = \frac{\sin x\cos y + \cos x\sin y}{\cos x\cos y - \sin x\sin y} \qquad (1)$$

into

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y} \tag{2}$$

by dividing both numerator and denominator on the right of equation (1) by $\cos x \cos y$, it should be noted that this process is valid only when

$$\cos x \neq 0 \text{ and } \cos y \neq 0.$$

Teachers sometimes have a wrong impression in regard to the determination of what is called the "true value" of an indeterminate expression. Such an expression as

$$\frac{x^2-4}{x-2}$$

has no determinate value when $x=2$. There is no such thing as a true value of this expression when $x=2$, except by definition.

An interesting case of indeterminate forms comes up in spheric trigonometry, in trying to determine the remaining parts of a spheric triangle when the three given parts are $a=90°$, $b=90°$, $A=90°$. The law of sines gives $B=90°$.

If we use these values in the formula

$$\tan\frac{c}{2} = \frac{\cos\frac{1}{2}(A+B)}{\cos\frac{1}{2}(A-B)} \cdot \tan\tfrac{1}{2}(a+b),$$

we find

$$\tan\frac{c}{2} = \frac{0}{1} \cdot \infty,$$

which is indeterminate, just as it should be, that is, the remaining parts cannot be determined; in fact c and C may have any value.

It is hoped that this brief article will suffice to call attention to the dangers involved in trigonometric operations on account of the possible presence of indeterminate expressions. Many teachers feel that indeterminates should not be taught in elementary mathematics; but unfortunately the teacher occasionally finds himself forced to take up the topic, whether he wishes to or not.

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THE COURTIS TESTS IN ARITHMETIC.

By Philip A. Boyer.

The efficiency of an operation is determined in large measure by the ability to evaluate results. In no department of educational work are there greater opportunities for service to the individual pupil than in the scientific measurement of the product. The rapid multiplication of tests and measurements in the field of education is at once an indication of their usefulness and a guarantee of their success in directing the search light of scrutiny to the minutiae of the teaching process. Indeed before one gets very far with such investigation he finds himself launched upon numerous related and more detailed studies, the sum total of which cannot fail to bring about a more intelligently purposeful contact between teacher and pupil.

Teachers test pupils religiously and even more devoutly do they detest the scoring of papers. This is due entirely to the barrenness of the task. Give to the test a scientifically determined purpose; tie it up intimately with specific pedagogic method; let it show to both teacher and pupil the precise degree of success attained in a very definite detail of subject matter; let this result be exactly comparable, not only with like scores from many other classes working under similar conditions, but also with the demands of society for certain and sure command of particular definitely useful details of knowledge; and the teacher is stolid indeed who does not thrill at the opportunity for real service. Testing of this kind furnishes a basis for the diagnosis of class and individual needs and suggests the immediate application of specific devices for improvement.

The general field of mathematics furnishes abundant material especially well adapted to purposeful testing, and the fundamental operations in arithmetic are peculiarly fitted to such treatment because they comprise a tool subject of definitely limited content, a subject which every child must know completely, use with automatic precision and in which rate is as

essential as accuracy. From the many standard tests and scales in the field of arithmetic the concensus of opinion of educators points to the selection of a general test of achievement in the fundamentals as the type of test best suited for preliminary survey. The results of such tests can be secured and tabulated with comparative ease and may be referred to scientifically determined and generally accepted standards. Such results will indicate many possibilities for further investigation and experiment and it is in this sort of work that the more elaborate and detailed tests and scales will be found useful.

The Courtis Standard Research Tests in Arithmetic Series B were chosen as best fitting the above conditions and were given in seven Philadelphia Schools in March, 1918, with the following five purposes in view:

1. To establish a criterion of judgment as to the comparative success of the present teaching of the fundamentals.

2. To find a base from which to measure future progress.

3. To provide for the development of definite, detailed and objective aims of instruction and drill in each grade, *i. e.*, to establish reasonable standards of achievement.

4. To furnish means for a diagnosis of the teaching of the fundamental operations in order to develop methods whereby the efficiency of instruction might be increased.

5. To bring to the attention of teachers the striking individual differences existing among pupils of any given class and to attempt to fit instruction and drill to the varying needs of such pupils.

Because of the very limited scope of the tests the attention of teachers could not fail to be drawn to the most minute elements of success and failure. The results were tabulated with enthusiasm and even before comparisons could be made there were attempts on every hand to relate achievements to specific processes of teaching and drill. In the very natural search for explanations both the teaching method and the elements of the operations themselves were carefully analyzed,—a spirit of scientific inquiry was instituted.

Comparisons of results were then made between classes, schools, and with the results obtained in other cities, as well as with the Courtis general medians and the Courtis standards.

Such comparisons are indeed interesting, but they must be made, as Mr. Courtis insists, with extreme caution. He says: "One should be careful to recognize that a score in a given test represents merely a *performance under the given conditions.* Every one should take pains to give and score the tests under standard conditions, but at best should expect to get from city-to-city comparisons only conclusions as to the nature and amount of relative progress and not judgements as to absolute achievements." The conditions mentioned include such things as time allowance, physical conditions, temperature, humidity and lighting. It should, therefore, be noted here that the tests were given to the group under consideration on dark, dismal and spiritless days. They were entirely new and strange to the great majority of pupils, and there must therefore have been at least a slight degree of tacit misunderstanding on the part of some pupils even with the most careful adherence on the part of the examiners to the standard directions of Mr. Courtis. Again, in Test No. 4 (division), the examples were arranged in a way wholly foreign to Philadelphia pupils. Even though the pupils were instructed to place the quotient to the right of the dividend, as was their custom, the lack of sufficient space for such quotient and the disconcerting vinculum above the dividend must have been disturbing factors. It should also be remembered that the scores here presented are March scores and as such are more nearly mid-year scores than the May or June scores with which they are compared. This condition alone would warrant us in expecting the scores to be lower than Standard June scores by at least one third of the grade interval in each case. These variant conditions will have to be kept in mind when comparisons are made with future results in the same schools and also when we draw comparisons with the results attained in other cities.

Class, grade and school medians have been tabulated for purposes of internal comparison, but only the median scores of the total pupilage of the seven schools tested will be presented here as of general interest. These are given in Table I, and will be herein referred to as the Philadelphia March medians. The relation of these medians to the Courtis General medians may be studied here, though for purposes of general comparison the

relations are shown more clearly in Graphs I and II. In Graph I, representing rate of work, the Philadelphia District medians are indicated by light full lines; the Courtis medians are shown

TABLE I.

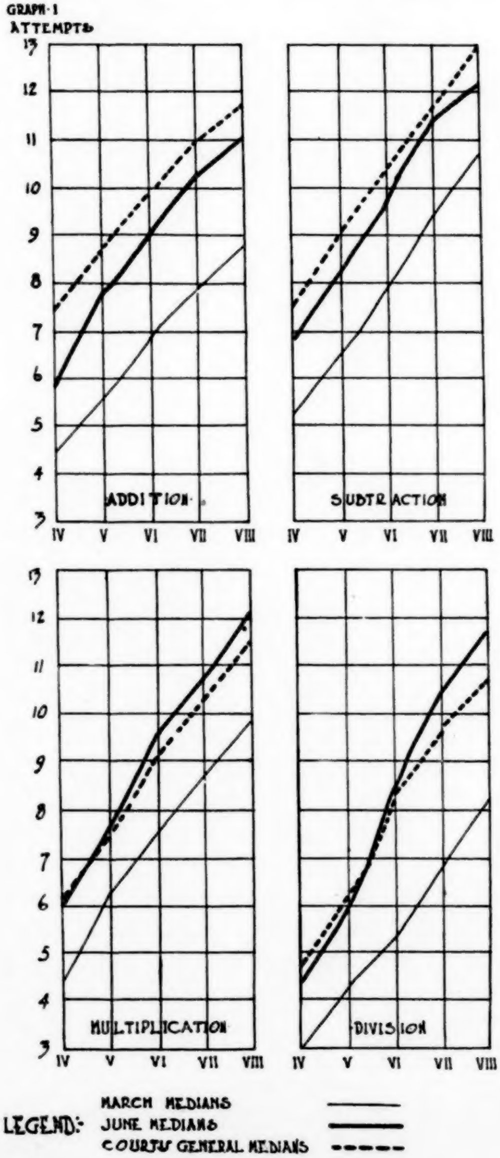
MEDIAN SCORES. COURTIS STANDARD TESTS. ARITHMETIC SERIES B.

Grade.	Addition.		Subtraction.		Multiplication.		Division.	
	Rate.*	Accuracy.*	Rate.	Accuracy.	Rate.	Accuracy.	Rate.	Accuracy.
IV. Phila. March Scores...	4.4	57	5.2	59	4.4	57	3.0	39
Prov. June Standards..	6	64	6	80	6	67	4	57
Phila. June Scores	5.8	64	6.8	82	6.0	69	4.3	67
Courtis Gen. Medians .	7.4	64	7.4	80	6.2	67	4.6	57
V. Phila. March Scores ...	5.4	64	6.5	77	6.3	72	4.2	59
Prov. June Standards .	7	70	8	83	7	75	5.5	[illegible]7
Phila. June Scores	7.8	70	8.2	84	7.6	78	6	82
Courtis Gen. Medians .	8.6	70	9	83	7.5	85	6.1	77
VI. Phila. March Scores...	6.9	72	7.8	82	7.6	78	5.2	77
Prov. June Standards .	8.5	73	9.5	85	8.5	78	7	87
Phila. June Scores	8.9	74	9.5	89	9.5	84	8.3	88
Courtis Gen. Medians .	9.8	73	10.3	85	9.1	78	8.2	87
VII. Phila. March Scores...	7.8	71	9.4	84	8.6	82	6.8	84
Prov. June Standards .	9.5	75	11	36	9.5	80	8.5	90
Phila. June Scores	10.3	78	11.4	90	10.6	85	10.3	93
Courtis Gen. Medians .	10.9	75	11.6	86	10.2	80	9.6	90
VIII. Phila. March Scores...	8.7	77	10.6	88	9.7	85	8.1	91
Prov. June Standards .	10.5	76	12	87	10.5	81	9.5	91
Phila. June Scores	11	78	12.1	92	12	88	11.6	99
Courtis Gen. Medians .	11.6	76	12.9	87	11.5	81	10.7	91

* Rate is here indicated in number of examples completed; accuracy in per cents.

by the broken lines. It will be seen at a glance that the Philadelphia scores show a progress almost exactly parallel with that indicated by the Courtis medians, but this is rather cold comfort when it is noted that Philadelphia medians run three attempts less than the Courtis medians in addition, two attempts less in subtraction and division, and more than one less in multiplication. The Courtis tests do not explain situations, do not diagnose; they merely state facts, the interpretation of which often calls for extended investigation, and it was with this sort of investigation that the Philadelphia committee proposed to occupy itself. The Philadelphia rate of work must be increased, not

·MEDIAN·NUMBER OF EXAMPLES·ATTEMPTED·



GRAPH I.

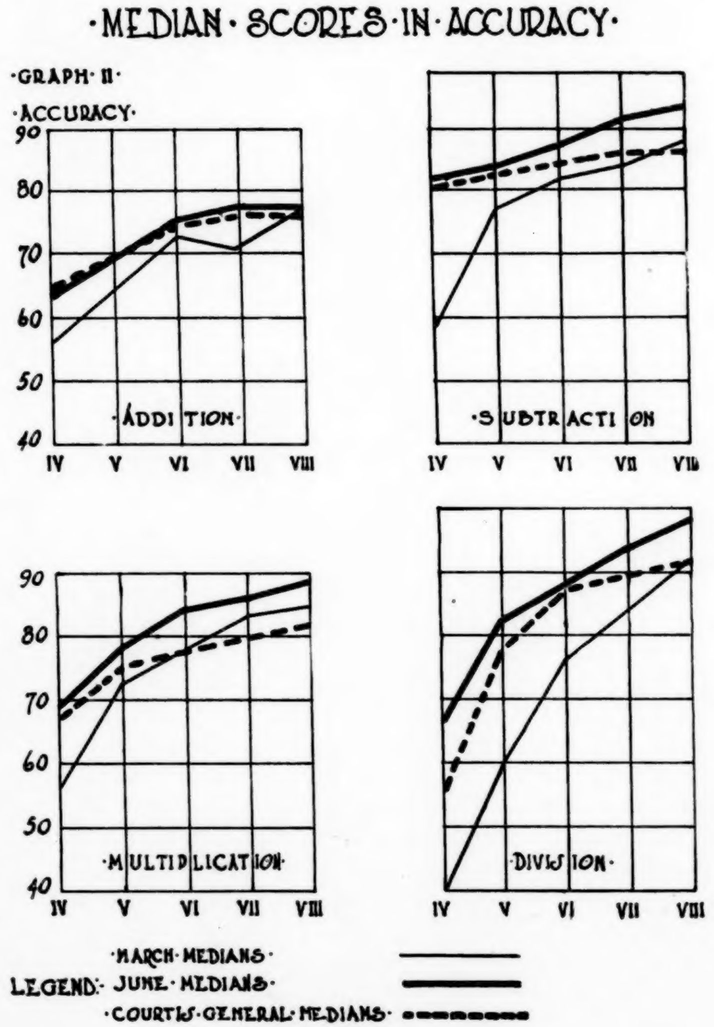
through hurry or "speed," but rather through careful revision of teaching method to bring about automatization of reactions.

In Graph II, which represents accuracy medians, we find a closer approximation of the Philadelphia medians to those of Courtis but only in the seventh grade in multiplication and in the eighth grade in all operations do they exceed those medians. It will be noted in this graph in all the operations and especially in subtraction that we do not find the same parallelism of the Philadelphia and Courtis results, as was the case in rate. The greater difference in grade four would seem to indicate that we are not developing proficiency in the fundamentals sufficiently early in the grades. In view of the necessity for constant use of these processes in the arithmetic work of the upper grades, the question may be raised whether it would not be more efficient to develop the fundamentals early. This, again, was a problem for the committee to study in detail. It may be found that the great progress from grade four to grade five ought to be moved down to between grades three and four, or rather that we should bring about in grade three what is now achieved in grade four.

Probably the most important result of the comparisons of the Philadelphia and Courtis results came from the establishment of definite and detailed aims for the work of each grade. These aims were set approximately midway between March achievements and the Courtis medians, for it did not seem reasonable to expect pupils to attain Courtis median proficiency in a period of three months. Presumably attainable goals were accordingly set up for each grade in each operation. These goals were to be reached in June when another test was promised.* The procedure here outlined is somewhat arbitrary to be sure but it established working aims which furnished compelling incentive to persistent and constructive endeavor on the part of pupils and teachers alike. Instead of looking forward with dread to the possibility of failing to make an "average" of 70 in some loosely determined and vague "examination," fourth-grade children, for example, knew that they would be expected to work in eight minutes six addition examples of a given type with sixty per cent. accuracy. Each child knew where he stood at the time and what he must work for, and

* See Provisional June Standards—Table I.

teachers were only too willing to assist in the process. From week to week the process of the class toward these definite



GRAPH II.

goals was shown graphically and individual pupils were encouraged to graph their own progress. The finest kind of

rivalry—that with one's own past achievements—was set up for each class and each individual.

As was pointed out in the above discussion both individual and class scores showed particular successes and failures. These were symptoms which required analysis, diagnosis and specific treatment. Ideally, of course, each individual should have instruction peculiar to his special need but with the pressure of many other kinds of work this is a practical impossibility in classes of forty to fifty pupils unless one of the few good devices for individual practice is used. These were available to only a small percentage of the classes under consideration.

Hence after a study of results in relation to class room procedure and methods of drill, the committee in charge of the work attempted to standardize the conditions. Each class was divided, according to achievement in each of the four fundamentals, into two groups. The smaller of these groups was made up of pupils who did notably poor work in the operation concerned. An attempt was made to keep this group small enough to permit of individual attention on the part of the teacher. The larger and better group had its drill as a class exercise. A time limit of eight minutes per day was set for drill with each group so that each teacher spent sixteen minutes per day in this work and each pupil spent eight minutes. Because of the change in the form of division to that used in the Courtis tests, two days, Monday and Friday, were given over to division. The three remaining operations were studied on Tuesday, Wednesday and Thursday respectively. In regard to method, the attention of teachers was called to the difficulty encountered in bridging the tens and to the time-wasting practice of saying, either audibly or inaudibly, the numbers of a given combination instead of automatically producing the answer. This practice definitely allied itself with silent reading. Zero difficulties, trial divisor difficulties, and others were analyzed, and suggestions as to specific methods and devices were presented.

To impress teachers further with the futility of uniform class drill, diagrams were prepared showing the wide range of variation in the attainments of the pupils of each grade in each operation. The diagram for only one of the operations (addition)

is shown here in Chart III. The portion outlined in solid line represents the spread of achievements in March and may be read as follows: 1 per cent. of eighth-grade pupils attempted 2

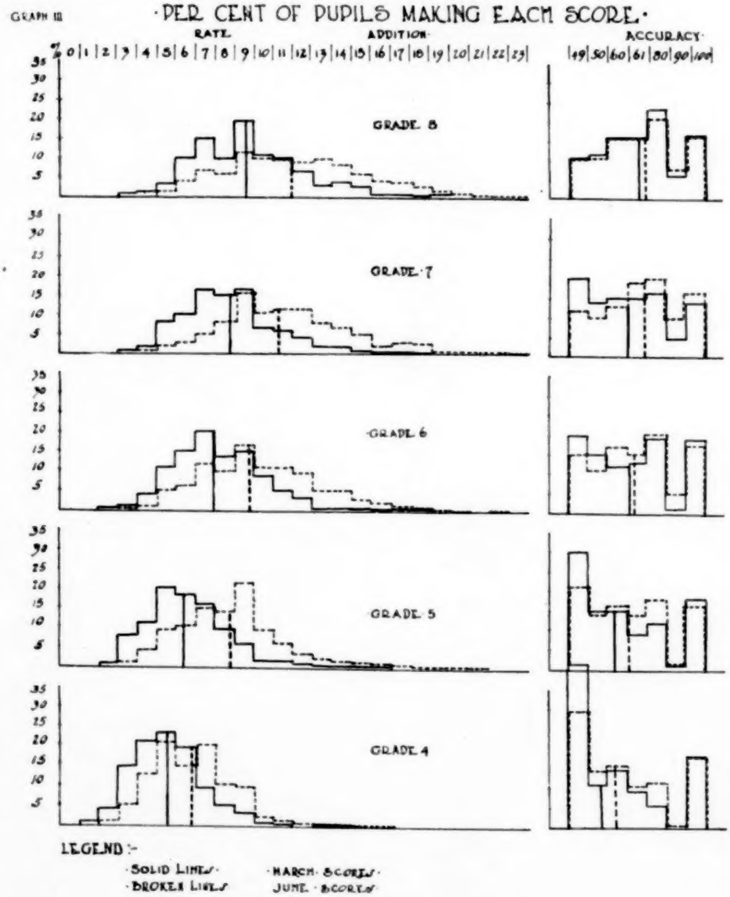


CHART III.

additions; 2 per cent. attempted 3; 3 per cent. attempted 4; 10 per cent. attempted 5 examples and so on to 1 per cent. who attempted 19 examples. The full vertical lines represent March medians, and it will be seen that there is a regular progress from grade to grade, but the total progress from grades 4 to 8 is

small compared with the great range of ability within a single grade. The exceptional cases are far too numerous to be neglected. Note for example, that about 3 per cent. of fourth-grade pupils attempt more examples than do half of the eighth-grade pupils, and that about 3 per cent. of eighth-grade pupils attempt fewer examples than do half of the fourth-grade pupils. The amount of individual variation or spread of achievement increases as we go up through the grades indicating that class methods are adapted to the average or brighter pupils at the expense of those at the lower end of the range.

At the right of this chart is indicated the spread of accuracy scores. The ideal achievement, of course, is 100 per cent. accuracy. It will be interesting to note that grades 4, 5 and 6 show a somewhat higher per cent. of perfect accuracy than do grades 7 and 8, and in no case except that of division is there a constant increase in the number of perfect scores through the grades. There is, however, in every case a constant decrease through the grades in the per cent. of scores showing less than 50 per cent. accuracy, and a general tendency is evident for the per cent. of higher accuracies to increase with advancing grades. The small number of 90 per cent. accuracies is explained by the fact that a pupil must attempt 10 examples before this score becomes possible.

After less than three months of special drill in the manner and under the time limits described, Form 2 of the same series of tests was administered. The June tests were given under more favorable conditions generally than those which prevailed in March. The weather was favorable, the pupils had been preparing for just such a test, they were accustomed to its arrangement and conditions and the form of the division operation was now familiar. Accordingly a marked improvement was anticipated. The results of this June test are given in Table I as June Medians and are indicated graphically by the heavy full line in Graphs I and II.

Examination of Graph I will show that the improvement in median rate of work for June over that of March was roughly two examples in addition, one and one half examples in subtraction, two examples in multiplication and $1\frac{1}{2}$ to $3\frac{1}{2}$ examples in division. It is interesting to note in multiplication and espe-

cially in division the gradually increasing improvement in rate as we ascend through the grades. Addition and subtraction rates fall short of the Courtis Medians by about ½ example; multiplication and division rates equal or slightly exceed the Courtis medians. In every case the June medians exceed the Provisional June standards by from ¼ to 1 example.

The definite aim of the special drill on the fundamentals was to increase rate of work without diminishing accuracy. Provisional June standards for accuracy were therefore not advanced materially over March achievements. Examination of Chart II shows that the drill nevertheless did increase accuracy even at the higher rate of work. June accuracy medians equal or exceed those of Courtis. It appears that increased rate means more complete automatization and therefore produces greater accuracy.

A further purpose of the special daily eight minute drill on fundamentals was to reduce the great range of variation in achievements within a given grade. It was for this reason that classes were divided into groups according to ability and special individual attention given to the weaker group. An indication of the success or failure of this proceedure may be studied in Graph III where the total spread of achievements in addition is diagramed for both March and June. It will be noted that June scores (in broken lines) show a wider range of variation in attempts in every grade than do the scores of March. Notwithstanding the fact that the pupils who did the better work in March were given only class drill, it is they who seem to have profited most by the exercise. Here, it seems, is sufficient warrant for eliminating a large part of the drill with pupils who have reached or exceeded the Courtis medians. These pupils should be free to engage in other work more appropriate to their immediate needs. While there is a noticeable reduction in the number of pupils who continue to make poor scores in June, practically all of the poorer grades of attainment have some representatives. These poorer pupils are the "problems" of the class room. Even individual treatment has failed to reach them in three months of drill. Some may lack ability, but most of them gradually will achieve success as the result of more persistent and continued effort. In accuracy, the June scores

show a more even spread and there is less variability than in the March scores.

If we take the quartile deviations for these addition scores we have a crude though satisfactory measure of their comparative variation. Table II shows that June variations are larger in rate and smaller in accuracy (except eighth-grade accuracy)

TABLE II.

QUARTILE DEVIATIONS (Q) IN ADDITION SCORES FOR MARCH AND JUNE.

Grade.	Rate.		Accuracy.	
	March.	June.	March.	June.
IV	1.2	1.5	25.0	20.7
V	1.3	1.5	27.0	16.5
VI	1.5	2.0	17.3	15.0
VII	1 4	2.2	16.2	15.3
VIII	2.0	2.6	13.7	13.9

than the corresponding March variations, which fact would seem to indicate that pupils who fail to make gains in rate tend to compensate by increased accuracy. This assumption will be borne out by inspection of Graph III where the lower per cents of accuracy are shown to be materially reduced. It is interesting to note further in Table II that quartile deviations in rate increase through the grades while deviations in accuracy decrease. Pupils of the lower grades have a rather uniform rate of work and show very diverse accuracy scores. As practice continues, the rate of work becomes more diverse and accuracy tends to become uniform. This condition would indicate that in the lower grades accuracy should be emphasized even at the expense of rate while in the higher grades more and more emphasis should be placed upon standard rates of work.

STANTON SCHOOL,
PHILADELPHIA, PA.

REPORT OF THE COMMITTEE TO RECOMMEND A SUITABLE PROGRAM IN MATHEMATICS FOR THE JUNIOR HIGH SCHOOL.

This committee was appointed by the president of the Association of Teachers of Mathematics in New England at the spring meeting, May 1, 1917. A preliminary meeting of the committee was held on May 15, 1917. The committee, now seven in all, has members from two large general high schools, one public technical high school, one public grammar school, and two private preparatory schools; it also has as a member one assistant superintendent of schools.

The committee held eight meetings at intervals of about four weeks, and one meeting in October, 1918. At no meeting were there more than two members absent.

The definition of the junior high school which the committee adopted was the three years commonly called the seventh, eighth and ninth, thus including what is now the first high school grade.

The deliberations of the committee may be divided into four parts.

I. The discussion and adoption of the aims of the mathematical course of the junior high school.

II. The discussion of topics for such a course, and the justification of each topic as a factor in realizing these aims.

III. The formation of these topics into a logical and teachable course.

IV. The discussion of the limitations and the interpretation of the meanings of the topics.

I. Aims.

After an extended discussion of the question, the committee adopted five general or cultural aims and five specific or utility aims, as follows:

A. General or Cultural Aims.

1. To develop habits of concise, exact, and logical thinking and expression.

2. To develop self reliance.
3. To develop a sense of personal responsibility.
4. To develop an ability to apply general principles to new problems.
5. To inculcate some appreciation of the influence which mathematics has had on the great sciences and industries of the world.

B. Specific or Utility Aims.

1. To produce accuracy in computation.
2. To produce reasonable speed in computation without sacrificing accuracy.
3. To develop a definite idea of number values.
4. To develop an ability to recognize the degree of accuracy possible with measured data.
5. To develop the ability to handle a variety of mathematical tools.

The committee feels that the definite nature of the three years covered by this course when considered as a period in the child's school career, makes it especially desirable that a program be planned that shall have a great degree of unity and very definite aims. These three years come at a time when many children are planning to leave school in a few years at most. The unity and completeness of the mathematical program will have its effect in keeping the child in school until the end of these three years. Hence an attempt has been made to formulate aims the accomplishment of which can be reasonably hoped for from a well-organized course in the beginnings of mathematics.

The general or cultural aims are statements of five general qualities which it should be the purpose of any course in elementary mathematics to develop. The specific or utility aims are the statements of five qualities of mathematical ability which the mathematical course of these particular three years should be expected to develop in the pupil.

The much debated question of whether any general abilities can be developed through the study of a particular subject need not be settled in order to justify these general aims. For our purposes, the widely accepted conclusion that acquired abilities can be transferred "with some loss" to fields in which similar elements exist is a sufficient reason for hoping to accomplish

these aims. We do not hope to develop habits of concise, exact, and logical thinking and expression for all children in all lines of thinking. We do not hope to develop self reliance or a sense of personal responsibility for every child in all of his fields of activity. If the training which he receives from his mathematical studies increases his ability to think logically and rely on himself in a few related fields, we are justified in keeping these qualities prominently before the teacher by formulating them as aims the accomplishment of which is desirable. The application of general principles to new problems and the appreciation of the influence which mathematics has had in the world's development are more specifically the aims of mathematical teaching than the first three, but are placed among the general aims because their accomplishment depends upon the general mathematical course rather than upon the specific topics covered in the junior high school program.

The specific aims give definite formulation to those mathematical abilities which the junior high school can be reasonably expected to train. Since the recent investigations and measurements of arithmetical ability in the schools show that accuracy is largely due to practice, we may justly expect the junior high school to aim to produce a fair degree of accuracy in computation. And since these same investigations show that speed is largely acquired through maturity coupled with practice, we feel justified in making the second specific aim "reasonable speed in computation without sacrificing accuracy." The development of a definite idea of number values will, it is hoped, enable the pupil to appreciate the difference between the height of an ordinary dwelling house and the height of a down-town business block. This ability to attach the proper approximate number values to the things of everyday life is an important one to develop before the pupil leaves school; or, if he is to continue in school, to develop at an early stage of his mathematical training. The ability to appreciate the degree of accuracy possible in measured data has been left very much in the background in early mathematical courses. Most of us, even yet, feel a little guilty when we use three and one seventh instead of 3.14159 in computing the number of feet of lumber in a round table. The

term "significant figures" is, perhaps, the reason for some of the fear prevalent in attacking the subject of measured data; but there can be no doubt that the simple idea involved in the fact that the answer cannot tell exactly how many thousandths of square inches we have when the original data told only the number of whole inches in the dimensions, is an important idea in the applications of mathematics and one which is within the grasp of the junior high school pupil. The term "significant figures" need not be used. The only important thing to be made clear is that there is a limit to the accuracy which can be obtained in any measurement made with a material instrument.

II. Discssion of Topics.

The committee decided that the following topics should appear in the junior high school course in mathematics.

Opposite each topic is placed a letter and number, or several of these, to indicate the aims which can be furthered by the study of that topic. It will be seen that each topic is thus justified by at least three aims.

First Year.

1. Review of fundamental operations of arithmetic. (A2, A3, A4, B1, B2.)
2. Measurement of straight lines, angles, and plane rectilinear figures. Drawing to scale. Straight line graphs. (A5, B3, B5.)
3. Equations: (*a*) $bx=c$; (*b*) $\frac{x}{a}=b$; (*c*) $\frac{b}{c}=\frac{x}{a}$ (ratio). (A1, A2, A3, B5.)
4. Percentage: (*a*) Finding what part one number is of another;
   (*b*) Finding percents of given amounts;
   (*c*) Applications—single discount, simple interest, commission. (A1, A4, A5, B1, B5.)
5. Mensuration: (*a*) Formulas for the areas of rectilinear figures and circles;

(*b*) Rectangular and circular graphs. (A4, A5, B1, B3, B5.)

Second Year.

1. Review of fundamental operations of arithmetic applied to business transactions. (A2, A3, A4, B1, B2.)
2. Percentage: (*a*) Finding base, percentage and rate given.
   (*b*) Applications—successive discounts, interest, notes, savings banks. (A4, A5, B1, B2, B5.)
3. Arithmetic of the home, of the farm, and of civic life. Taxes and insurance. (A3, A4, A5, B5.)
4. Plane geometrical figures—construction and classification of lines, angles, triangles, and quadrilaterals; graphical representation of statistics using squared paper. (A4, A5, B5.)
5. Formulas: (*a*) Perimeters and areas of plane figures including a review of equations.
   (*b*) Square root, Pythagorean theorem.
   (*c*) Surfaces and volumes of solids. (A2, A4, A5, B5.)
6. Equations: (*a*) $ax \pm b = c$, $ax \pm b = cx \pm d$.
   (*b*) Pairs of linear equations. Graphs. (A1, A2, A3, B5.)

Third Year.

1. Percentage (profit on selling price, per cent. error). Appriximate computation (square root). (A3, A4, B3, B4.)
2. Construction and evaluation of formulas. Metric measurements. (A1, A5, B5.)
3. Linear equations (review of types of second year, applications from geometry). (A1, A5, B1.)
4. Algebraic expressions (positive and negative numbers, addition and subtraction of polynomials, parenthases). (A1, A5, B5.)
5. Multiplication and division of algebraic expressions (monomials and binomials for multipliers and divisors). (A1, A5, B5.)

6. Factoring: $am + bm + cm$; $a^2 - b^2$; $a^2 \pm 2ab + c^2$; $a^2 \pm 2ab + b^2 - c^2$; $x^2 + bx + c$; $ax^2 + bx + c$. (A1, A2, B5.)
7. Quadratic equations: (*a*) Solution by factoring. Applications from geometry.
   (*b*) Solution by completing the square. (A1, A4, B4, B5.)
8. Ratio (ratios of line segments, or areas, of volumes, sine ratio and tangent ratio). Variation (simple problems in direct variation). (A4, A5, B5.)
9. Variables—Pairs of equations.
   (*a*) Graphs of linear pairs.
   (*b*) Solution of linear pairs by addition and subtraction.
   (*c*) Graphs of linear-quadratic pairs.
   (*d*) Solution of linear-quadratic pairs. (A1, A5, B5.)
10. Fractional equations (monomial and binomial denominators of first degree). (A1, A4, B5.)
11. Properties of plane geometric figures informally developed (lines, angles, triangles, and quadrilaterals). (A1, A4, B5.)

III. Arrangement of Topics.

After several rearrangements of the above topics, it was decided that the order here given was a suitable teaching order. Although the committee does not feel that the order adopted is the only order possible for these topics, it does feel convinced that the purpose of the course can be most effectively carried out by this order, and that any material change would be liable to defeat the orderly development of the whole plan as a unit.

IV. Limitations and Interpretation of Topics.

The topics as given in the outline may suggest to some teachers more than is really intended by the committee. A few cases of such limitations may be mentioned.

1. Each of the four fundamental operations of arithmetic should be checked.

2. Numbers involving decimals should not, as a rule, be carried beyond the third decimal place, or, if significant figures are used, not beyond the fourth significant figure.

3. Mental estimates of results should be made before the results are computed.

4. There should be no abstract problems in denominate numbers.

5. Selling price as a basis of figuring per cent. profit is used by business houses, and the committee strongly recommends this method.

6. No discussion of stocks and bonds nor compound interest should be given.

7. The treatment of discount should not involve the reduction of several discounts to a single discount.

8. The treatment of taxes should be limited to the finding of taxes on property.

9. Problems in insurance should involve finding the premium only.

10. Per cent. errors should be computed from measured and from counted numbers.

11. In the second year, linear pairs should be limited to equations not involving negative numbers.

12. We recommend that in problems involving the removal of parentheses, only one parenthesis within another he used.

13. We recommend the use of a table of square roots for evaluating the roots of quadratic equations.

14. We also recommend the following methods: Horizontal addition to be used from the beginning; Newton's method of extracting square roots.

The committee is aware that the program here laid out will require teachers well trained in mathematics. It must be remembered, however, that the junior high school is to contain the first present high school grade, and that in order to offer this grade instruction of as high an order as it is now getting, we must provide teachers as well trained as those now teaching in this grade. If this is to be done by resorting to the normal schools for the supply, these schools must offer training in mathematics beyond one year of high school algebra which is now the limit of this training in most normal schools. If, however, the school committees and superintendents will insist upon having college trained teachers for the junior high schools, the

problem will be solved, and we can hope to have the program here outlined effectively carried out.

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* Died December 19, 1918.

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BOOK REVIEWS.

Modern Arithmetic. By BRUCE M. WATSON and CHARLES E. WHITE. New York: D. C. Heath and Company. For *Upper Grades*, p. 302; *Intermediate*, p. 254; *Primary*, p. 252.

These books not only lay out a complete and well-arranged course for the pupils, but they are also very suggestive to the teacher. The authors evidently realize the possibilities of games, relating the work to industries and other parts of the school's environment, and creating still other motives for arithmetical work.

They emphasize drill, but so vary it with thought work and applications that the teacher can hardly become merely a taskmaster.

The books merit examination by those introducing new texts in arithmetic.

Unified Mathematics. By L. C. KARPINSKI, H. Y. BENEDICT and I. W. CALHOUN. Boston: D. C. Hunt & Co. Pp. 522. $2.80.

This new textbook in "Combined Mathematics" is particularly suitable for the courses of the S. A. T. C. because of its emphasis on: Computation—from the very beginning stress is laid upon computation, and the use of logarithms is continued throughout the work; Trigonometry—the essentials of trigonometry are treated thoroughly but concisely; Graphical Methods—constant practice is given in the use of coördinate paper such as is employed by engineers and at the proving grounds; Applications—particular attention is paid to problems dealing with projectiles, and the "mil," the artillery unit of angular measurement, is carefully explained.

Four Place Logarithmic and Trigonometric Tables, with Interest Tables. By LOUIS C. KARPINSKI. Ann Arbor, Mich.: George Wahr. Pp. 30. Price 30 cts.

These four-place tables are intended to increase speed and accuracy. They seem well devised for this purpose as the arrangement and type are both good. The size of the book makes it convenient to handle, and its price is low enough to make it available to many whose need does not justify an expensive set of tables.

Introductory Algebra Exercises. By WILLIAM BETZ. Published by the Board of Education, Rochester, N. Y. Pp. 73. Price 40 cts.

Mr. Betz has given us in the very interesting little book the introductory part of the algebra course as it has been worked out in the Rochester high schools. The book has many unusual and excellent features, but the introduction to the subject in which he explains why algebra is to be studied, is perhaps its best point.

NOTES AND NEWS.

THOSE who have been helped by THE MATHEMATICS TEACHER will be interested to know that Dr. William H. Metzler, who has been editor-in-chief since the magazine was started, has been called to France to take an executive part in organizing the educational work for our soldiers overseas.

Dr. Metzler is the one who by his enthusiasm and devotion has made this magazine possible. To it he has given unsparingly of his time and his thought, and he has many times accomplished what seemed to his co-workers impossibilities.

While he will be greatly missed, the magazine, like its compatriots, is willing to give its best to its country's work.

Dr. Metzler has asked the other editors to carry on his work during his absence, and the main office of the magazine in Syracuse will be kept open. Whatever concerns the subject-matter may be sent directly to Eugene R. Smith, The Park School, Baltimore, Md., and anything concerning the advertising may be sent to William E. Breckenridge, Stuyvesant High School, New York City.

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"SUCCESSFUL FARMING," Des Moines, Iowa, is furnishing free to those teachers who ask for it a monthly "Rural Schools Bulletin." The bulletins contain many suggestions for use in rural community schools.

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School Life, published twice a month by the Bureau of Education, should be in every school. The current issues contain a great deal of interesting information about educational matters.

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"HEY, THERE! Do you want a Home on a Farm?" is the title of a booklet issued by the Department of the Interior for soldiers and sailors. It tells how any man who has been wearing the United States uniform can apply for a chance to own his own farm.

The Committee of Special War Activities of the National Catholic War Council has published an interesting pamphlet called "Social Reconstruction," reviewing the problems before us, and making a survey of possible remedies for some of the existing conditions.

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The National Industrial Conference Board in a recent bulletin attack the "Lessons in Community and National Life," issued by the United States Bureau of Education, under the title "A Case of Federal Propaganda in Our Public Schools."

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Continuation Schools are one of the most interesting of recent educational developments. Swift and Company of Chicago have outlined the aims and procedure in their school in a prospectus which they will furnish to those who are interested.